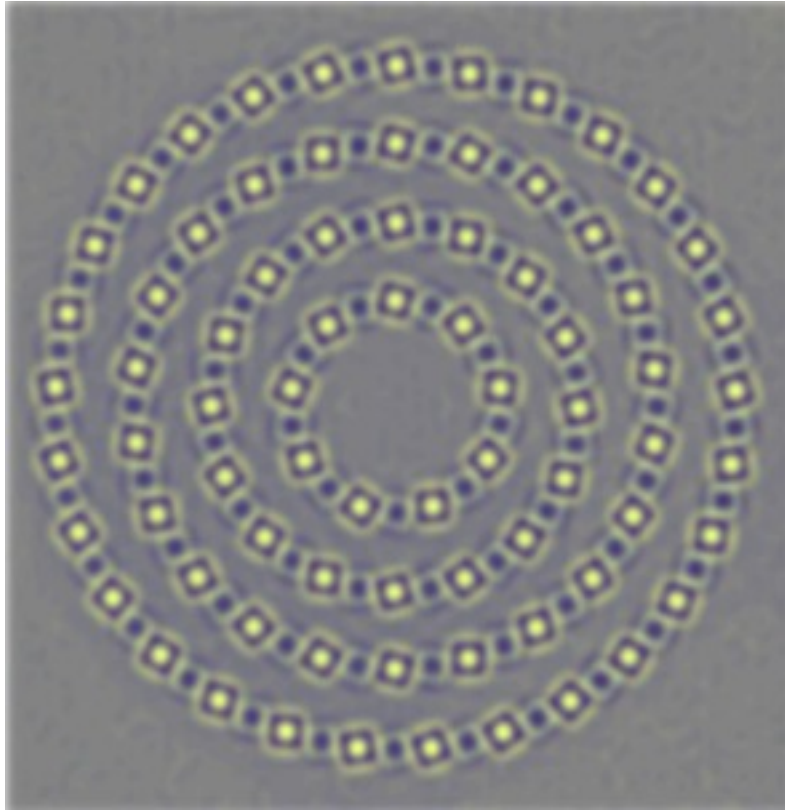# Optical Illusions Essay

Angela Wall
EMAT 6690

Optical illusions are images that are visually perceived differently than how they actually appear in reality. These images can be very entertaining, but also very deceiving. Optical illusions are caused by either an error with one's visual sense or an error of intellect. This "error of intellect" occurs when the visual senses correctly interprets an image, but the mind's imagination, false assumptions, or inexperiences causes the illusion. Visual illusions with different effects can be found in numerous places, challenging the interests of scientists, artists, and artisans. Artist M. C. Escher utilized several optical illusions in his works, baffling the minds of mathematicians, scientists, and all other viewers. Optical illusions play an important role in our appreciation of both the physical world and the perceived physical world. Many optical illusions are very mathematical and geometric in nature. Implementing activities in the classroom involving such types of visual illusions could be intellectually stimulating, mathematically enriching, and thought-provoking for many students. The illusions described and explained below could be presented to students at the secondary level when studying several geometric concepts. For each illusion below, the instructions for constructing the illusion in The Geometer's Sketchpad (GSP) follows each explanation. All images below with the exception of the three dimensional Poggendorff image and the stereokinetic phenomenon image were constructed using GSP.

Stereokinetic Phenomenon

Background Information/History:
The prefix "stereo" comes from the Greek meaning "solid." It is used with reference to hardness, solidity, and three-dimensionality in the formation of compound words. Kinetic describes an object in motion. Thus, as the name might suggest, stereokinetic phenomenon are three-dimensional objects assembled by two-dimensional shapes in slow rotatory motion, making them appear solid. They were first investigate by C. L. Musatti in the 1920s. Commonly produced by circles, stereokinetic phenomenon can also be created using ellipses, rectangles, and bars forming ellipsoids, cylinders, and other three-dimensional objects.

Explanations and Mathematical Connections:
The phenomena formed appears to have a defined length in depth. Through calculations of velocity and time, the depth of the three-dimension illusion can be predicted. The phenomenon is caused by the minimization of velocity differences. Our visual system minimizes relative velocity differences between the various points of the pattern, determining the apparent height of the three-dimensional object formed. When just looking at the image from a two-dimensional angle, however, it is merely just a set of circles. The circles are fixed to one of two different points that lie on the same radius of the biggest circle. In the image above, half the circles are tangent to the same point on the outer circle and the other half of the circles are tangent to a point close to the center of the largest circle. The two points are then animated to rotate to cause the illusion.

Application for the Classroom and Constructing the Illusion in GSP:
The illusion of the stereokinetic phenomenon fits naturally with a discussion of circles in the classroom. The different circles in the image could be given different radii values and students could use them to find the equation for each circle. Students could also use their knowledge of circles to find different areas, circumferences, and even the area of circular disks.

GSP Sketch

The construction of the illusion in GSP is not too difficult of a task. After starting with a circle, the diameter of the circle was drawn. Using one of the points of intersection of the diameter and the circle, the original circle was dilated a couple times about that point. The point of intersection between the last dilated circle and the original circle's diameter was then found. A circle was constructed tangent to this point that was smaller than the previous circles. This circle was then dilated to be smaller a few times. The point on the outer circle was then animated to rotate around the circle.

To See in Motion: http://www.michaelbach.de/ot/mot_ske/index.html

The Hering Illusion and Its Variations

Background Information/History:
The original Hering Illusion was discovered by psychologist Ewald Hering in the 1860s. Today, there are several different variations of the original Hering Illusion, but they all focus on the illusion of straight lines appearing bowed or bent. The construction and placement of other lines affects the viewer's depth perception, creating a false impression. The Hering Illusion has been used to aid neuroscientists study the basics of perception and they have been used by certain artists in their works. The illusion is best seen when the lines that appear bowed are either horizontal or vertical. When the lines are at an angle, the illusion is not as effective. This concept has questioned some of the traditional theories on the generation of the illusory effect.
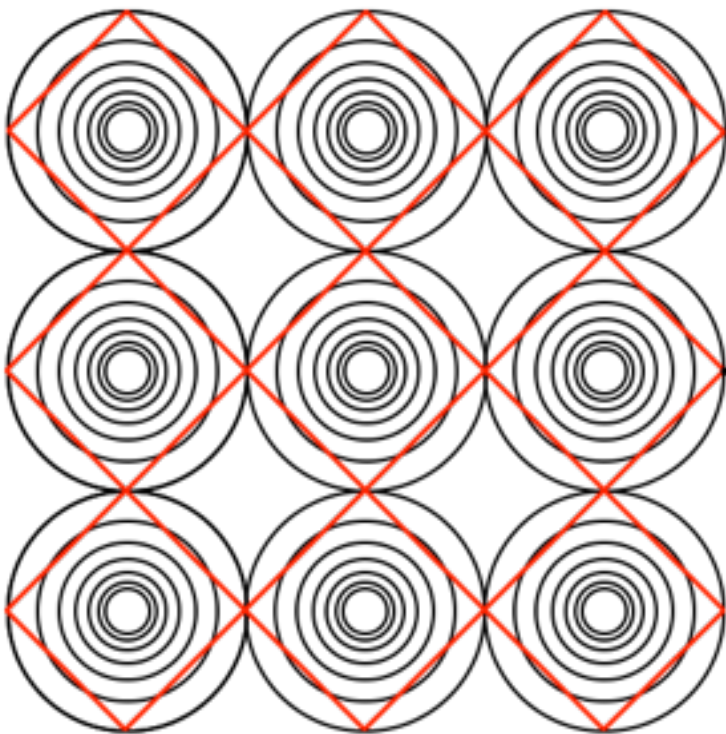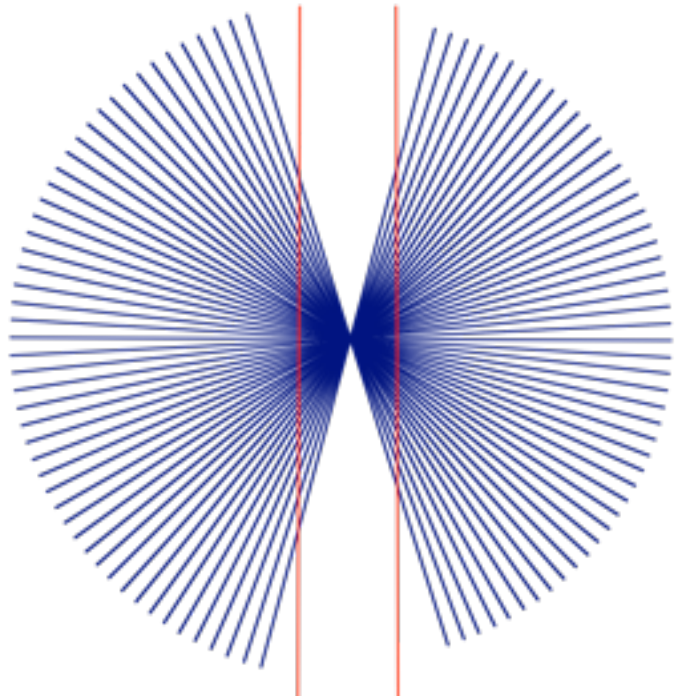
Explanations and Mathematical Connections:
The Hering Illusion can be connected to several geometric concepts. The perception of the parallel lines of the original illusion is skewed because of the multiple lines in the background that are rotated at different angles about a fixed point. The same is true for the lines in the other two variations of the illusion. Some believe that the lines appear to bow out because the figures in the background effect one's depth perception. The central spot of the figures in the background is perceived as being further away than around the edges. This effect tricks the brain into thinking that the lines are more widely spaced out at the center of the background image. Several illusions, including the Hering Illusion, has proved to scientists that people do not always see the physical world. The brain takes information we see and simplifies it into something that is more meaningful. When it comes to optical illusions, the brain is mistaken.
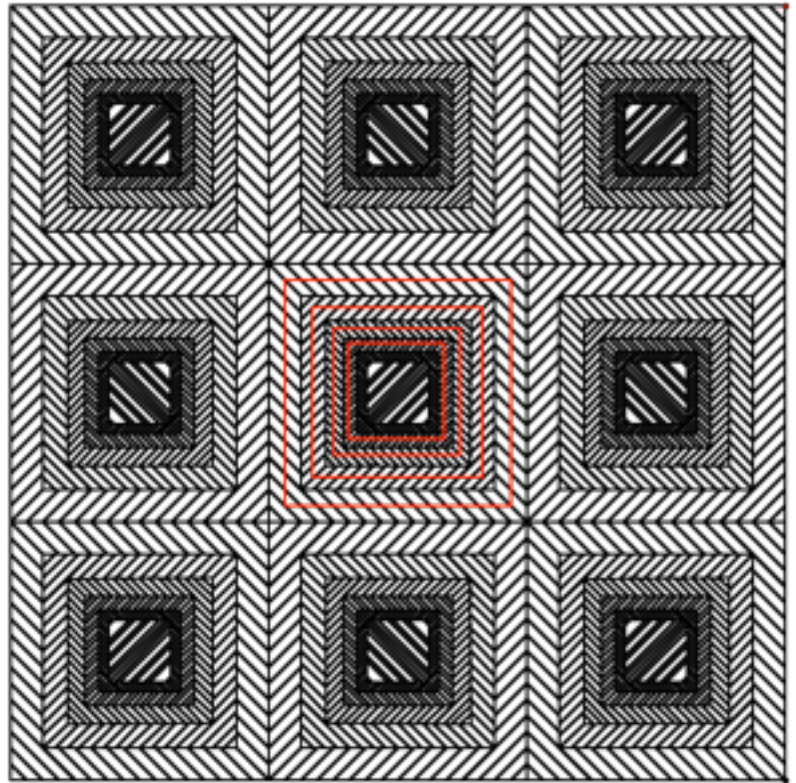
The Hering Illusion and its different variations could be used when discussing several concepts in the classroom. The original Hering Illusion an be used to discuss the notion of angles. The background image is just multiple lines rotated at different angles. This could start a discussion about the number of degrees in a circle, the degree of which to rotate lines to construct a similar image, and even the Unit Circle. The original illusion could even be connected to parallel lines cut by a transversal and the Vertical Angles Theorem. Students could even prove that the lines are parallel using angles and the properties of parallel lines cut by a transversal.

The second version of the Hering Illusion incorporates circles in the background. This would allow students to explore the basics of a circle, such as radius, diameter, area, and circumference. It could also be used to discuss the connection between lines and circles. Students could discuss chords, tangent lines, and secant lines with this illusion. The notion of an angle inscribed in a circle could also be addressed with the image.

The third version of the Hering Illusion could be used to discuss transformations. Students could explore how each layer or border of the squares are rotated and dilated to create the next inside layer. Students could also explore the shapes and designs made by the diagonal lines in each layer. Values could be given to the lengths of some of the squares and they could be used to find the areas of certain squares. For instance, the area of the one large square could be found, the area of any of the smaller squares, or the area of the borders around squares.

GSP Sketches

Although the Hering Illusion might appear complex to the mind, the construction of the illusion and its variations can be easily accomplished in GSP. Students would be very capable of completing such a construction when studying transformations. The construction of the original Hering Illusion in GSP was not too difficult. It started with a horizontal line which was then rotated by 3 degrees 25 times. The original horizontal line was marked as a mirror and then all the rotated lines were selected and reflected across the marked mirror. A red vertical line was then drawn through the left side of the "fan" of lines, somewhat close to the center of rotation. A vertical line was then drawn through the center of rotation and marked as a mirror. The red line was reflected across the mirror and the mirror was then hidden. This is just one of several ways the illusion could have been constructed using GSP.

The second Hering Illusion above required several different transformations in GSP. After starting with a constructed circle, the circle was dilated by a factor of 3/4. The dilated circle was then dilated by the same factor. This process was repeated until there were about six or seven circles total. A vertical line was drawn through the center of the circles and a point was made where the line intersected the biggest circle. Through that point, another line was drawn perpendicular to the first line. This line was marked as a mirror. All the circles were then selected and reflected across the mirror. The entire process was repeated until there was a column of three sets of circles. A similar process was completed to reflect the column of circles across a vertical mirror twice. After completing reflections and hiding mirrors, the set of circles was 3 X 3. To construct the lines in the illusion, the points of intersection between all the larger circles were found. In other words, on each of the larger circles, there was a point at North, South, East, and West. The points were connected to make multiple squares. This process, however, could have been done before all the circles were reflected to make the 3 X 3.

The final Hering Illusion discussed was the most intricate construction in GSP. After constructing one square, one of its diagonals was drawn. The midpoint of the diagonal was found, and then the midpoint of one half of the diagonal was found. This process was repeated until the diagonal was cut into 32 equal parts. Through these points, lines were constructed that were perpendicular to the diagonal, and then the diagonal was hidden. Another square was constructed within the first square and all segments were removed within the second square. In other words, a square disk with diagonal lines within the area of the disk was formed (See image to the right for clarification). After making the square disk, several transformations were required to make the next disk. The entire disk was selected, reflected across a mirror parallel to the disk, dilated, rotated 90 degrees, and reflected back to get the next border in the original square. The same process was repeated until five square disks were created. The middle of the square was filled in with lines parallel to to the lines from the two previous iterations. All unnecessary transformations were hid throughout the process. When one of the square sections was completed, the entire thing was reflected several times vertically and horizontally to form a 3 X 3 square of squares. In the middle square, red squares were constructed within each layer to form the illusion.
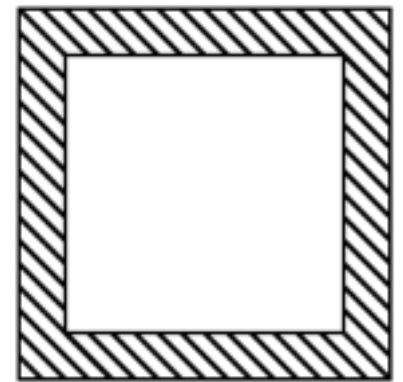
The Zöllner Illusion
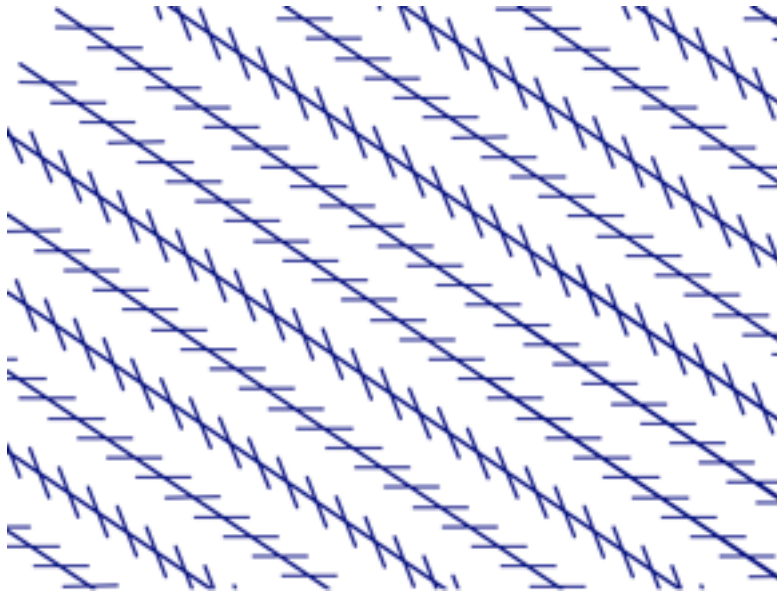
Background Information/History:
The Zöllner Illusion was discovered by astrophysicist Johann Zöllner in 1860 when studying the pattern on a piece of cloth. Intrigued by the illusion, he shared his findings with J. C. Poggendorff who subsequently discovered the Poggendorff Illusion using some of the observations from the Zöllner Illusion. The Zöllner Illusion, Poggendorff Illusion, and Hering Illusion are all closely related because they all demonstrate how lines can appear distorted because of the images in the background.

Explanations and Mathematical Connections:
The Zöllner Illusion contains a set of parallel, oblique lines crossed with short, angled line segments. The oblique lines do not appear parallel, when they are in reality. On explanation of the illusion suggests that the angle of the shorter line segments compared to the longer lines create an impression of depth making some lines appear closer than others. Another explanation suggests that the brain tries to increase the angles between the short and long lines causing the lines to bend away and towards each other, resulting in the illusion. The illusion caused seems to increase when the angle between the shorter and longer lines is between 10 and 30 degrees. Interestingly, when the background of the illusion is either red or green instead of white, the illusion is no longer visible.

Application for the Classroom and Constructing the Illusion in GSP:
Like most of the optical illusions discussed, the Zöllner Illusion also involves several geometric concepts and applications. Since the image uses several parallel lines, the concept of parallel lines and slope can be discussed using this illusion. The slope of the lines that appear nonparallel can be found to prove that the lines are in fact parallel. Transformations could also be discussed using this optical illusion. The students could try to figure out which transformations could be used to construct the illusion.

GSP Sketch

The most tedious part of constructing this illusion in GSP is constructing a line with several intersecting small segments with the same slope and the same distance apart. This was done by drawing a line with evenly spaced points across the line (done by constructing several line segments and midpoints along the line). Two parallel lines were then placed on both sides of the original line, equidistant from the original line. These two parallel lines were used with the several points to make the short line segments along the original line. As soon as the original line had all of its short line segments, then entire construction was reflected across a mirror parallel to the original line. This was done numerous times to get the final outcome. The small line segments along the original line could have also been constructed by rotating the original line, dilating it to shorten it, and then translating it for the repetition.
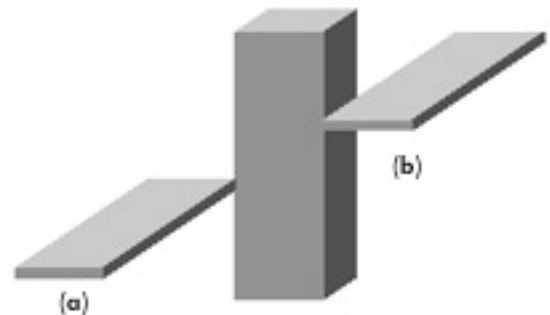
The Poggendorff Illusion

Background Information/History:
Similar to the Hering Illusion, the Poggendorff Illusion distorts the brain's perception of certain objects. The illusion was discovered by physicist J.C. Poggendorff in 1860 after studying the illusion discovered by Zöllner. While studying the Zöllner Illusion, Poggendorff observed and determined another illusion, what is now known as the Poggendorff Illusion. The illusion is cause by two ends of a line passing behind a rectangle that appear offset, when they are in fact aligned. The Poggendorff Illusion is similar to the Hering and Zöllner Illusion in that the background image obscures the perception of lines.

Explanations and Mathematical Connections:
The Poggendorff Illusion contains only a set of parallel lines passing behind a set of rectangles of the same size and evenly spaced apart. The angles of the parallel lines and the width of the rectangle also has some effect on the illusion. As the slope of the parallel lines gets steeper, the effect of the illusion increases. The largest displacement of the parallel lines appear when the parallel lines are nearly vertical and the rectangles are at maximum width. There are two popular theories for the explanation of the Poggendorff Illusion. The angular displacement theory suggests that the brain minimizes all obtuse angles and exaggerates all acute angles. The contour of the rectangle in the image causes the eyes to respond differently to the different angles, thus causing the illusion. The second theory is the depth-processing, or consistency, theory. This theory suggests that the rectangles and lines in the image are viewed as three dimensional objects in a plane. Like with the angular displacement theory, the acute angles are overestimated and the obtuse angles are underestimated. The misperception of the angle causes the illusion. The misperception, however, is not caused by a two-dimensional distortion, but by perspective representation. See the image above for a three dimensional representation of the Poggendorff Illusion.

Application for the Classroom and Constructing the Illusion in GSP:
The Poggendorff Illusion could also be explored in the classroom when discussing different geometric concepts. One exploration could be with slope and parallel lines. Students could complete an activity showing that line segments that appear offset are in fact part of the same line. To do this, a piece of the image could be placed on the coordinate plane. The students could find the slope of the two line segments to show that they are parallel. To show that they are in fact the same line, the students would have to find slope using two points, one on either side of the rectangle. The value for the slope should be the same as the slope of the two line segments on either side of the rectangle. This illusion could also be used with an activity and exploration with transformations. Ask for student input on how to construct the illusion in as few steps as possible by using transformations. With this illusion, several reflections and translations could be used.

GSP Sketch

There are several methods that can be used to construct the illusion using GSP. For the image above, first, several parallel lines were made by translating one line numerous times. The lines were then shortened into segments to form a rectangle. In the GSP sketch, the rectangles in the image can change widths using an adjustable line segment. The separate adjustable line segment was used to make the rectangles. It was translated on top of parallel lines and then used to construct a rectangle. The constructed rectangle was then translated several times horizontally to cover all the parallel lines.

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